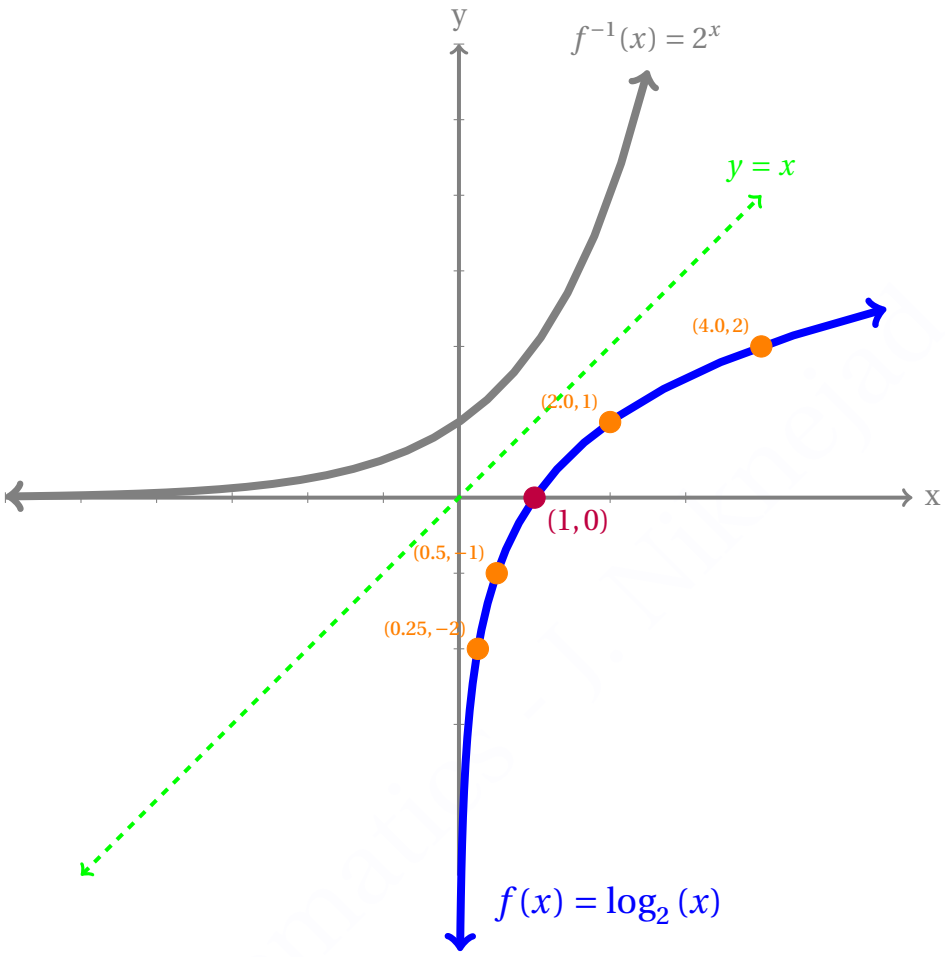
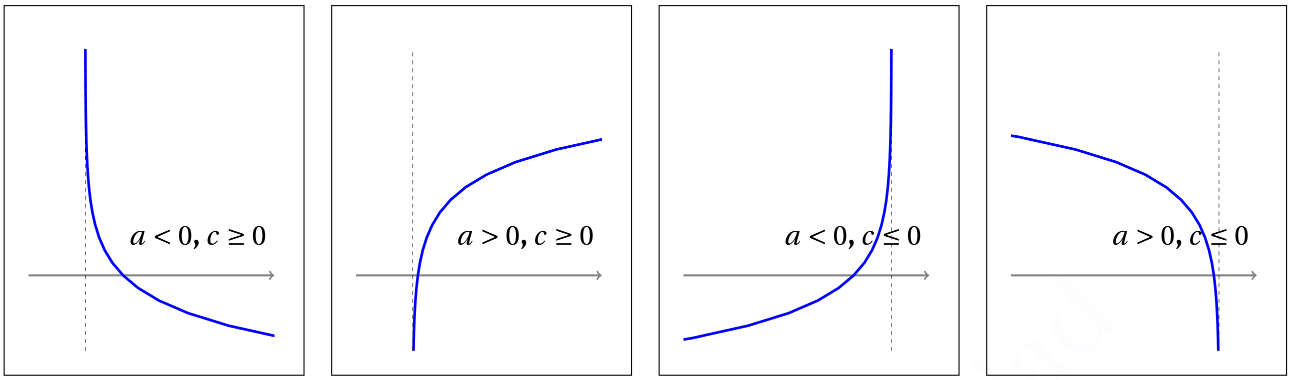
4.4: Graph of Logarithmic Function



- Graph of a logarithmic function $f(x) = \log_b(x)$ is reflection of its inverse $y = b^x$ over $y = x$.
- The logarithmic function only has a vertical asymptote and one x-intercept.
- Function $f(x) = a\log_b(cx+d)+f$ is obtained from $g(x) = \log_b(x)$ by a horizontal shift of $|d|$ units, possibly a reflection over y-axis if $c < 0$, a horizontal shrinking/stretching of ratio $|c|$, a possible reflection over x-axis if $a < 0$, a shrinking/stretching of ratio $|a|$ and a vertical shift of f.
- Position of vertical asymptote for $f(x) = a\log_b(\underbrace{cx+d}_{=0}) + f$ is $x = -\dfrac{d}{c}$.
- Position of the vertical asymptotes for any log function can be found by setting inside of the log equal to zero.
- Position of x-intercept for any log function can be found by setting the function equal to zero. For a function of the form $f(x) = \log(g(x))$, set the inside function equal to one. See Section 4.6 for finding the x-intercepts of logarithmic functions.
- Domain of log function can be found by setting $\underbrace{\text{inside of the log}}_{>0}$ strictly bigger than zero and solving the resulting inequality.

- The general shape of a logarithmic function $f(x) = a\log_b(cx + d) +$ is



1. Consider the function $f(x) = 2\ln(x + 3) + 1$.

   (A) Find the function's x-intercept.

   (B) Find its asymptotes.

   (C) Find its domain.

   (D) Graph the function.

2. Consider the function $f(x) = \ln\left(2x^2 - 3x + 1\right)$.

   (A) Find its asymptote(s).

   (B) Find its domain.

   (C) Find the x-intercept(s).

3. Let $f(x) = \log_5(x - 1) + 7$. Then $f^{-1}(x) =$

   (A) $\ln(x - 7) + 1$

   (B) $5x - 2$

   (C) $5^{x-7} + 1$

   (D) $\dfrac{x - 7}{5} + 1$